

# CONTENTS

<b>PREFACE TO THE FIFTH EDITION</b>	<b>xix</b>
<b>PREFACE TO THE FOURTH EDITION</b>	<b>xxiii</b>
<b>PREFACE TO THE THIRD EDITION</b>	<b>xxv</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Five Important Practical Problems, 2	
1.1.1 Forecasting Time Series, 2	
1.1.2 Estimation of Transfer Functions, 3	
1.1.3 Analysis of Effects of Unusual Intervention Events to a System, 4	
1.1.4 Analysis of Multivariate Time Series, 4	
1.1.5 Discrete Control Systems, 5	
1.2 Stochastic and Deterministic Dynamic Mathematical Models, 6	
1.2.1 Stationary and Nonstationary Stochastic Models for Forecasting and Control, 7	
1.2.2 Transfer Function Models, 11	
1.2.3 Models for Discrete Control Systems, 13	
1.3 Basic Ideas in Model Building, 14	
1.3.1 Parsimony, 14	
1.3.2 Iterative Stages in the Selection of a Model, 15	
Appendix A1.1 Use of the R Software, 17	
Exercises, 18	

- 3.4.1 Stationarity and Invertibility Properties, 75
- 3.4.2 Autocorrelation Function and Spectrum of Mixed Processes, 77
- 3.4.3 First Order Autoregressive First-Order Moving Average Process, 78
- 3.4.4 Summary, 81

Appendix A3.1 Autocovariances, Autocovariance Generating Function, and Stationarity Conditions for a General Linear Process, 82

Appendix A3.2 Recursive Method for Calculating Estimates of Autoregressive Parameters, 84

Exercises, 86

## 4 Linear Nonstationary Models 88

- 4.1 Autoregressive Integrated Moving Average Processes, 88
  - 4.1.1 Nonstationary First-Order Autoregressive Process, 88
  - 4.1.2 General Model for a Nonstationary Process Exhibiting Homogeneity, 90
  - 4.1.3 General Form of the ARIMA Model, 94
- 4.2 Three Explicit Forms for the ARIMA Model, 97
  - 4.2.1 Difference Equation Form of the Model, 97
  - 4.2.2 Random Shock Form of the Model, 98
  - 4.2.3 Inverted Form of the Model, 103
- 4.3 Integrated Moving Average Processes, 106
  - 4.3.1 Integrated Moving Average Process of Order (0, 1, 1), 107
  - 4.3.2 Integrated Moving Average Process of Order (0, 2, 2), 110
  - 4.3.3 General Integrated Moving Average Process of Order (0,  $d$ ,  $q$ ), 114

Appendix A4.1 Linear Difference Equations, 116

Appendix A4.2 IMA(0, 1, 1) Process with Deterministic Drift, 121

Appendix A4.3 ARIMA Processes with Added Noise, 122

- A4.3.1 Sum of Two Independent Moving Average Processes, 122
- A4.3.2 Effect of Added Noise on the General Model, 123
- A4.3.3 Example for an IMA(0, 1, 1) Process with Added White Noise, 124
- A4.3.4 Relation between the IMA(0, 1, 1) Process and a Random Walk, 125
- A4.3.5 Autocovariance Function of the General Model with Added Correlated Noise, 125

Exercises, 126

## 5 Forecasting 129

- 5.1 Minimum Mean Square Error Forecasts and Their Properties, 129
  - 5.1.1 Derivation of the Minimum Mean Square Error Forecasts, 131
  - 5.1.2 Three Basic Forms for the Forecast, 132
- 5.2 Calculating Forecasts and Probability Limits, 135
  - 5.2.1 Calculation of  $\psi$  Weights, 135
  - 5.2.2 Use of the  $\psi$  Weights in Updating the Forecasts, 136
  - 5.2.3 Calculation of the Probability Limits at Different Lead Times, 137
  - 5.2.4 Calculation of Forecasts Using  $R$ , 138
- 5.3 Forecast Function and Forecast Weights, 139

- 5.3.1 Eventual Forecast Function Determined by the Autoregressive Operator, 140
- 5.3.2 Role of the Moving Average Operator in Fixing the Initial Values, 140
- 5.3.3 Lead  $l$  Forecast Weights, 142
- 5.4 Examples of Forecast Functions and Their Updating, 144
  - 5.4.1 Forecasting an IMA(0, 1, 1) Process, 144
  - 5.4.2 Forecasting an IMA(0, 2, 2) Process, 147
  - 5.4.3 Forecasting a General IMA(0,  $d$ ,  $q$ ) Process, 149
  - 5.4.4 Forecasting Autoregressive Processes, 150
  - 5.4.5 Forecasting a (1, 0, 1) Process, 153
  - 5.4.6 Forecasting a (1, 1, 1) Process, 154
- 5.5 Use of State-Space Model Formulation for Exact Forecasting, 155
  - 5.5.1 State-Space Model Representation for the ARIMA Process, 155
  - 5.5.2 Kalman Filtering Relations for Use in Prediction, 157
  - 5.5.3 Smoothing Relations in the State Variable Model, 160
- 5.6 Summary, 162
- Appendix A5.1 Correlation Between Forecast Errors, 164
  - A5.1.1 Autocorrelation Function of Forecast Errors at Different Origins, 164
  - A5.1.2 Correlation Between Forecast Errors at the Same Origin with Different Lead Times, 165
- Appendix A5.2 Forecast Weights for any Lead Time, 166
- Appendix A5.3 Forecasting in Terms of the General Integrated Form, 168
  - A5.3.1 General Method of Obtaining the Integrated Form, 168
  - A5.3.2 Updating the General Integrated Form, 170
  - A5.3.3 Comparison with the Discounted Least-Squares Method, 171
- Exercises, 174

**PART TWO STOCHASTIC MODEL BUILDING 177**

**6 Model Identification 179**

- 6.1 Objectives of Identification, 179
  - 6.1.1 Stages in the Identification Procedure, 180
- 6.2 Identification Techniques, 180
  - 6.2.1 Use of the Autocorrelation and Partial Autocorrelation Functions in Identification, 180
  - 6.2.2 Standard Errors for Estimated Autocorrelations and Partial Autocorrelations, 183
  - 6.2.3 Identification of Models for Some Actual Time Series, 185
  - 6.2.4 Some Additional Model Identification Tools, 190
- 6.3 Initial Estimates for the Parameters, 194
  - 6.3.1 Uniqueness of Estimates Obtained from the Autocovariance Function, 194
  - 6.3.2 Initial Estimates for Moving Average Processes, 194
  - 6.3.3 Initial Estimates for Autoregressive Processes, 196

- 6.3.4 Initial Estimates for Mixed Autoregressive–Moving Average Processes, 197
- 6.3.5 Initial Estimate of Error Variance, 198
- 6.3.6 Approximate Standard Error for  $\bar{w}$ , 199
- 6.3.7 Choice Between Stationary and Nonstationary Models in Doubtful Cases, 200
- 6.4 Model Multiplicity, 202
  - 6.4.1 Multiplicity of Autoregressive–Moving Average Models, 202
  - 6.4.2 Multiple Moment Solutions for Moving Average Parameters, 204
  - 6.4.3 Use of the Backward Process to Determine Starting Values, 205
- Appendix A6.1 Expected Behavior of the Estimated Autocorrelation Function for a Nonstationary Process, 206
- Exercises, 207

## 7 Parameter Estimation 209

- 7.1 Study of the Likelihood and Sum-of-Squares Functions, 209
  - 7.1.1 Likelihood Function, 209
  - 7.1.2 Conditional Likelihood for an ARIMA Process, 210
  - 7.1.3 Choice of Starting Values for Conditional Calculation, 211
  - 7.1.4 Unconditional Likelihood, Sum-of-Squares Function, and Least-Squares Estimates, 213
  - 7.1.5 General Procedure for Calculating the Unconditional Sum of Squares, 216
  - 7.1.6 Graphical Study of the Sum-of-Squares Function, 218
  - 7.1.7 Examination of the Likelihood Function and Confidence Regions, 220
- 7.2 Nonlinear Estimation, 226
  - 7.2.1 General Method of Approach, 226
  - 7.2.2 Numerical Estimates of the Derivatives, 227
  - 7.2.3 Direct Evaluation of the Derivatives, 228
  - 7.2.4 General Least-Squares Algorithm for the Conditional Model, 229
  - 7.2.5 ARIMA Models Fitted to Series A–F, 231
  - 7.2.6 Large-Sample Information Matrices and Covariance Estimates, 233
- 7.3 Some Estimation Results for Specific Models, 236
  - 7.3.1 Autoregressive Processes, 236
  - 7.3.2 Moving Average Processes, 238
  - 7.3.3 Mixed Processes, 238
  - 7.3.4 Separation of Linear and Nonlinear Components in Estimation, 239
  - 7.3.5 Parameter Redundancy, 240
- 7.4 Likelihood Function Based on the State-Space Model, 242
- 7.5 Estimation Using Bayes' Theorem, 245
  - 7.5.1 Bayes' Theorem, 245
  - 7.5.2 Bayesian Estimation of Parameters, 246
  - 7.5.3 Autoregressive Processes, 247
  - 7.5.4 Moving Average Processes, 249
  - 7.5.5 Mixed Processes, 250
- Appendix A7.1 Review of Normal Distribution Theory, 251

A7.1.1	Partitioning of a Positive-Definite Quadratic Form, 251	
A7.1.2	Two Useful Integrals, 252	
A7.1.3	Normal Distribution, 253	
A7.1.4	Student's $t$ Distribution, 255	
Appendix A7.2	Review of Linear Least-Squares Theory, 256	
A7.2.1	Normal Equations and Least Squares, 256	
A7.2.2	Estimation of Error Variance, 257	
A7.2.3	Covariance Matrix of Least-Squares Estimates, 257	
A7.2.4	Confidence Regions, 257	
A7.2.5	Correlated Errors, 258	
Appendix A7.3	Exact Likelihood Function for Moving Average and Mixed Processes, 259	
Appendix A7.4	Exact Likelihood Function for an Autoregressive Process, 266	
Appendix A7.5	Asymptotic Distribution of Estimators for Autoregressive Models, 274	
Appendix A7.6	Examples of the Effect of Parameter Estimation Errors on Variances of Forecast Errors and Probability Limits for Forecasts, 277	
Appendix A7.7	Special Note on Estimation of Moving Average Parameters, 280	
Exercises,	280	
<b>8</b>	<b>Model Diagnostic Checking</b>	<b>284</b>
8.1	Checking the Stochastic Model, 284	
8.1.1	General Philosophy, 284	
8.1.2	Overfitting, 285	
8.2	Diagnostic Checks Applied to Residuals, 287	
8.2.1	Autocorrelation Check, 287	
8.2.2	Portmanteau Lack-of-Fit Test, 289	
8.2.3	Model Inadequacy Arising from Changes in Parameter Values, 294	
8.2.4	Score Tests for Model Checking, 295	
8.2.5	Cumulative Periodogram Check, 297	
8.3	Use of Residuals to Modify the Model, 301	
8.3.1	Nature of the Correlations in the Residuals When an Incorrect Model Is Used, 301	
8.3.2	Use of Residuals to Modify the Model, 302	
Exercises,	303	
<b>9</b>	<b>Analysis of Seasonal Time Series</b>	<b>305</b>
9.1	Parsimonious Models for Seasonal Time Series, 305	
9.1.1	Fitting Versus Forecasting, 306	
9.1.2	Seasonal Models Involving Adaptive Sines and Cosines, 307	
9.1.3	General Multiplicative Seasonal Model, 308	
9.2	Representation of the Airline Data by a Multiplicative $(0, 1, 1) \times (0, 1, 1)_{12}$ Model, 310	
9.2.1	Multiplicative $(0, 1, 1) \times (0, 1, 1)_{12}$ Model, 310	
9.2.2	Forecasting, 311	
9.2.3	Model Identification, 318	
9.2.4	Parameter Estimation, 320	

9.2.5	Diagnostic Checking, 324	
9.3	Some Aspects of More General Seasonal ARIMA Models, 325	
9.3.1	Multiplicative and Nonmultiplicative Models, 325	
9.3.2	Model Identification, 327	
9.3.3	Parameter Estimation, 328	
9.3.4	Eventual Forecast Functions for Various Seasonal Models, 329	
9.3.5	Choice of Transformation, 331	
9.4	Structural Component Models and Deterministic Seasonal Components, 331	
9.4.1	Structural Component Time Series Models, 332	
9.4.2	Deterministic Seasonal and Trend Components and Common Factors, 335	
9.4.3	Estimation of Unobserved Components in Structural Models, 336	
9.5	Regression Models with Time Series Error Terms, 339	
9.5.1	Model Building, Estimation, and Forecasting Procedures for Regression Models, 340	
9.5.2	Restricted Maximum Likelihood Estimation for Regression Models, 344	
Appendix A9.1	Autocovariances for Some Seasonal Models, 345	
	Exercises, 349	
<b>10</b>	<b>Additional Topics and Extensions</b>	<b>352</b>
10.1	Tests for Unit Roots in ARIMA Models, 353	
10.1.1	Tests for Unit Roots in AR Models, 353	
10.1.2	Extensions of Unit Root Testing to Mixed ARIMA Models, 358	
10.2	Conditional Heteroscedastic Models, 361	
10.2.1	The ARCH Model, 362	
10.2.2	The GARCH Model, 366	
10.2.3	Model Building and Parameter Estimation, 367	
10.2.4	An Illustrative Example: Weekly S&P 500 Log Returns, 370	
10.2.5	Extensions of the ARCH and GARCH Models, 372	
10.2.6	Stochastic Volatility Models, 377	
10.3	Nonlinear Time Series Models, 377	
10.3.1	Classes of Nonlinear Models, 378	
10.3.2	Detection of Nonlinearity, 381	
10.3.3	An Empirical Example, 382	
10.4	Long Memory Time Series Processes, 385	
10.4.1	Fractionally Integrated Processes, 385	
10.4.2	Estimation of Parameters, 389	
	Exercises, 392	
<b>PART THREE</b>	<b>TRANSFER FUNCTION AND MULTIVARIATE MODEL BUILDING</b>	<b>395</b>
<b>11</b>	<b>Transfer Function Models</b>	<b>397</b>
11.1	Linear Transfer Function Models, 397	

11.1.1	Discrete Transfer Function, 398	
11.1.2	Continuous Dynamic Models Represented by Differential Equations, 400	
11.2	Discrete Dynamic Models Represented by Difference Equations, 404	
11.2.1	General Form of the Difference Equation, 404	
11.2.2	Nature of the Transfer Function, 406	
11.2.3	First- and Second-Order Discrete Transfer Function Models, 407	
11.2.4	Recursive Computation of Output for Any Input, 412	
11.2.5	Transfer Function Models with Added Noise, 413	
11.3	Relation Between Discrete and Continuous Models, 414	
11.3.1	Response to a Pulsed Input, 415	
11.3.2	Relationships for First- and Second-Order Coincident Systems, 417	
11.3.3	Approximating General Continuous Models by Discrete Models, 419	
Appendix A11.1	Continuous Models with Pulsed Inputs, 420	
Appendix A11.2	Nonlinear Transfer Functions and Linearization, 424	
Exercises,	426	
<b>12</b>	<b>Identification, Fitting, and Checking of Transfer Function Models</b>	<b>428</b>
12.1	Cross-Correlation Function, 429	
12.1.1	Properties of the Cross-Covariance and Cross-Correlation Functions, 429	
12.1.2	Estimation of the Cross-Covariance and Cross-Correlation Functions, 431	
12.1.3	Approximate Standard Errors of Cross-Correlation Estimates, 433	
12.2	Identification of Transfer Function Models, 435	
12.2.1	Identification of Transfer Function Models by Prewhitening the Input, 437	
12.2.2	Example of the Identification of a Transfer Function Model, 438	
12.2.3	Identification of the Noise Model, 442	
12.2.4	Some General Considerations in Identifying Transfer Function Models, 444	
12.3	Fitting and Checking Transfer Function Models, 446	
12.3.1	Conditional Sum-of-Squares Function, 446	
12.3.2	Nonlinear Estimation, 447	
12.3.3	Use of Residuals for Diagnostic Checking, 449	
12.3.4	Specific Checks Applied to the Residuals, 450	
12.4	Some Examples of Fitting and Checking Transfer Function Models, 453	
12.4.1	Fitting and Checking of the Gas Furnace Model, 453	
12.4.2	Simulated Example with Two Inputs, 458	
12.5	Forecasting with Transfer Function Models Using Leading Indicators, 461	
12.5.1	Minimum Mean Square Error Forecast, 461	
12.5.2	Forecast of CO <sub>2</sub> Output from Gas Furnace, 465	
12.5.3	Forecast of Nonstationary Sales Data Using a Leading Indicator, 468	

12.6	Some Aspects of the Design of Experiments to Estimate Transfer Functions, 469	
Appendix A12.1	Use of Cross-Spectral Analysis for Transfer Function Model Identification, 471	
A12.1.1	Identification of Single-Input Transfer Function Models, 471	
A12.1.2	Identification of Multiple-Input Transfer Function Models, 472	
Appendix A12.2	Choice of Input to Provide Optimal Parameter Estimates, 473	
A12.2.1	Design of Optimal Inputs for a Simple System, 473	
A12.2.2	Numerical Example, 476	
	Exercises, 477	
<b>13</b>	<b>Intervention Analysis, Outlier Detection, and Missing Values</b>	<b>481</b>
13.1	Intervention Analysis Methods, 481	
13.1.1	Models for Intervention Analysis, 481	
13.1.2	Example of Intervention Analysis, 484	
13.1.3	Nature of the MLE for a Simple Level Change Parameter Model, 485	
13.2	Outlier Analysis for Time Series, 488	
13.2.1	Models for Additive and Innovational Outliers, 488	
13.2.2	Estimation of Outlier Effect for Known Timing of the Outlier, 489	
13.2.3	Iterative Procedure for Outlier Detection, 491	
13.2.4	Examples of Analysis of Outliers, 492	
13.3	Estimation for ARMA Models with Missing Values, 495	
13.3.1	State-Space Model and Kalman Filter with Missing Values, 496	
13.3.2	Estimation of Missing Values of an ARMA Process, 498	
	Exercises, 502	
<b>14</b>	<b>Multivariate Time Series Analysis</b>	<b>505</b>
14.1	Stationary Multivariate Time Series, 506	
14.1.1	Cross-Covariance and Cross-Correlation Matrices, 506	
14.1.2	Covariance Stationarity, 507	
14.1.3	Vector White Noise Process, 507	
14.1.4	Moving Average Representation of a Stationary Vector Process, 508	
14.2	Vector Autoregressive Models, 509	
14.2.1	VAR( $p$ ) Model, 509	
14.2.2	Moment Equations and Yule–Walker Estimates, 510	
14.2.3	Special Case: VAR(1) Model, 511	
14.2.4	Numerical Example, 513	
14.2.5	Initial Model Building and Least-Squares Estimation for VAR Models, 515	
14.2.6	Parameter Estimation and Model Checking, 518	
14.2.7	An Empirical Example, 519	
14.3	Vector Moving Average Models, 524	
14.3.1	Vector MA( $q$ ) Model, 524	
14.3.2	Special Case: Vector MA(1) Model, 525	
14.3.3	Numerical Example, 525	



- 14.3.4 Model Building for Vector MA Models, 526
- 14.4 Vector Autoregressive–Moving Average Models, 527
  - 14.4.1 Stationarity and Invertibility Conditions, 527
  - 14.4.2 Covariance Matrix Properties of VARMA Processes, 528
  - 14.4.3 Nonuniqueness and Parameter Identifiability for VARMA Models, 528
  - 14.4.4 Model Specification for VARMA Processes, 529
  - 14.4.5 Estimation and Model Checking for VARMA Models, 532
  - 14.4.6 Relation of VARMA Models to Transfer Function and ARMAX Models, 533
- 14.5 Forecasting for Vector Autoregressive–Moving Average Processes, 534
  - 14.5.1 Calculation of Forecasts from ARMA Difference Equation, 534
  - 14.5.2 Forecasts from Infinite VMA Form and Properties of Forecast Errors, 536
- 14.6 State-Space Form of the VARMA Model, 536
- 14.7 Further Discussion of VARMA Model Specification, 539
  - 14.7.1 Kronecker Structure for VARMA Models, 539
  - 14.7.2 An Empirical Example, 543
  - 14.7.3 Partial Canonical Correlation Analysis for Reduced-Rank Structure, 545
- 14.8 Nonstationarity and Cointegration, 546
  - 14.8.1 Vector ARIMA Models, 546
  - 14.8.2 Cointegration in Nonstationary Vector Processes, 547
  - 14.8.3 Estimation and Inferences for Cointegrated VAR Models, 549
- Appendix A14.1 Spectral Characteristics and Linear Filtering Relations for Stationary Multivariate Processes, 552
  - A14.1.1 Spectral Characteristics for Stationary Multivariate Processes, 552
  - A14.1.2 Linear Filtering Relations for Stationary Multivariate Processes, 553
- Exercises, 554

## **PART FOUR DESIGN OF DISCRETE CONTROL SCHEMES 559**

### **15 Aspects of Process Control 561**

- 15.1 Process Monitoring and Process Adjustment, 562
  - 15.1.1 Process Monitoring, 562
  - 15.1.2 Process Adjustment, 564
- 15.2 Process Adjustment Using Feedback Control, 566
  - 15.2.1 Feedback Adjustment Chart, 567
  - 15.2.2 Modeling the Feedback Loop, 569
  - 15.2.3 Simple Models for Disturbances and Dynamics, 570
  - 15.2.4 General Minimum Mean Square Error Feedback Control Schemes, 573
  - 15.2.5 Manual Adjustment for Discrete Proportional–Integral Schemes, 575

15.2.6	Complementary Roles of Monitoring and Adjustment, 578
15.3	Excessive Adjustment Sometimes Required by MMSE Control, 580
15.3.1	Constrained Control, 581
15.4	Minimum Cost Control with Fixed Costs of Adjustment and Monitoring, 582
15.4.1	Bounded Adjustment Scheme for Fixed Adjustment Cost, 583
15.4.2	Indirect Approach for Obtaining a Bounded Adjustment Scheme, 584
15.4.3	Inclusion of the Cost of Monitoring, 585
15.5	Feedforward Control, 588
15.5.1	Feedforward Control to Minimize Mean Square Error at the Output, 588
15.5.2	An Example: Control of the Specific Gravity of an Intermediate Product, 591
15.5.3	Feedforward Control with Multiple Inputs, 593
15.5.4	Feedforward–Feedback Control, 594
15.5.5	Advantages and Disadvantages of Feedforward and Feedback Control, 596
15.5.6	Remarks on Fitting Transfer Function–Noise Models Using Operating Data, 597
15.6	Monitoring Values of Parameters of Forecasting and Feedback Adjustment Schemes, 599
Appendix A15.1	Feedback Control Schemes Where the Adjustment Variance Is Restricted, 600
A15.1.1	Derivation of Optimal Adjustment, 601
A15.1.2	Case Where $\delta$ Is Negligible, 603
Appendix A15.2	Choice of the Sampling Interval, 609
A15.2.1	Illustration of the Effect of Reducing Sampling Frequency, 610
A15.2.2	Sampling an IMA(0, 1, 1) Process, 610
	Exercises, 613

<b>PART FIVE</b>	<b>CHARTS AND TABLES</b>	<b>617</b>
	<b>COLLECTION OF TABLES AND CHARTS</b>	<b>619</b>
	<b>COLLECTION OF TIME SERIES USED FOR EXAMPLES IN THE TEXT AND IN EXERCISES</b>	<b>625</b>
	<b>REFERENCES</b>	<b>642</b>
	<b>INDEX</b>	<b>659</b>