## Contents

	Pref	ace	XV
I	INT	RODUCTION	1
1	Introduction		3
	1.1	What Economists Can Compute	3
	1.2	Roles of Computation in Economic Analysis	6
	1.3	Computation in Science	13
	1.4	Future of Computing	15
	1.5	Objectives and Nature of This Book	17
	1.6	Basic Mathematics, Notation, and Terminology	20
	1.7	Software and Supplemental Material	25
	1.8	Further Reading	26
		Exercises	27
2	Elen	entary Concepts in Numerical Analysis	29
	2.1	Computer Arithmetic	29
	2.2	Computer Processing and Algorithms	31
	2.3	Economics of Computation	33
	2.4	Efficient Polynomial Evaluation	34
	2.5	Efficient Computation of Derivatives	35
	2.6	Direct versus Iterative Methods	39
	2.7	Error: The Central Problem of Numerical Mathematics	39
	2.8	Making Infinite Sequences Finite	41
	2.9	Methods of Approximation	44
	2.10	Evaluating the Error in the Final Result	45
	2.11	Computational Complexity	48
	2.12	Further Reading and Summary	50
		Exercises	50
П	BAS	ICS FROM NUMERICAL ANALYSIS ON R <sup>n</sup>	53
3	Linea	ar Equations and Iterative Methods	55
	3.1	Gaussian Elimination, LU Decomposition	55
	3.2	Alternative Methods	58
	3.3	Banded Sparse Matrix Methods	61
	3.4	General Sparse Matrix Methods	62

	3.5	Error Analysis	66
	3.6	Iterative Methods	70
	3.7	Operator Splitting Approach	75
	3.8	Convergence of Iterative Schemes	77
	3.9	Acceleration and Stabilization Methods	78
	3.10	Calculating $A^{-1}$	84
	3.11	Computing Ergodic Distributions	85
	3.12	Overidentified Systems	88
	3.13	Software	88
	3.14	Further Reading and Summary	89
		Exercises	89
4	Optir	nization	93
	4.1	One-Dimensional Minimization	94
	4.2	Multidimensional Optimization: Comparison Methods	99
	4.3	Newton's Method for Multivariate Problems	103
	4.4	Direction Set Methods	109
	4.5	Nonlinear Least Squares	117
	4.6	Linear Programming	120
	4.7	Constrained Nonlinear Optimization	121
	4.8	Incentive Problems	128
	4.9	Computing Nash Equilibrium	133
	4.10	A Portfolio Problem	135
	4.11	A Simple Econometric Example	137
	4.12	A Dynamic Optimization Problem	140
	4.13	Software	142
	4.14	Further Reading and Summary	142
		Exercises	143
5	Nonl	inear Equations	147
	5.1	One-Dimensional Problems: Bisection	147
	5.2	One-Dimensional Problems: Newton's Method	150
	5.3	Special Methods for One-Dimensional Problems	158
	5.4	Elementary Methods for Multivariate Nonlinear Equations	159
	5.5	Newton's Method for Multivariate Equations	167
	5.6	Methods That Enhance Global Convergence	171
	5.7	Advantageous Transformations	174
		-	

Contents

	5.8	A Simple Continuation Method	176
	5.9	Homotopy Continuation Methods	179
	5.10	A Simple CGE Problem	187
	5.11	Software	191
	5.12	Further Reading and Summary	192
		Exercises	193
6	Аррі	roximation Methods	195
	6.1	Local Approximation Methods	195
	6.2	Ordinary Regression as Approximation	202
	6.3	Orthogonal Polynomials	203
	6.4	Least Squares Orthogonal Polynomial Approximation	207
	6.5	Uniform Approximation	211
	6.6	Interpolation	216
	6.7	Approximation through Interpolation and Regression	219
	6.8	Piecewise Polynomial Interpolation	224
	6.9	Splines	225
	6.10	Examples	228
	6.11	Shape-Preserving Approximation	231
	6.12	Multidimensional Approximation	235
	6.13	Finite Element Approximations	240
	6.14	Neural Networks	244
	6.15	Further Reading and Summary	247
		Exercises	248
-	Num	erical Integration and Differentiation	251
	7.1	Newton-Cotes Formulas	251
	7.2	Gaussian Formulas	257
	7.3	Singular Integrals	267
	7.4	Adaptive Quadrature	269
	7.5	Multidimensional Quadrature	269
	7.6	Example: Portfolio Problems	277
	7.7	Numerical Differentiation	279
	7.8	Software	282
	7.9	Further Reading and Summary	282
		Exercises	283

ix

8	Moi	ite Carlo and Simulation Methods	285
	8.1	Pseudorandom Number Generation	285
	8.2	Monte Carlo Integration	291
	8.3	Optimization by Stochastic Search	296
	8.4	Stochastic Approximation	301
	8.5	Standard Optimization Methods with Simulated Objectives	303
	8.6	Further Reading and Summary	305
		Exercises	306
9	Qua	si-Monte Carlo Methods	309
	9.1	Equidistributed Sequences	311
	9.2	Low-Discrepancy Methods	314
	9.3	Fourier Analytic Methods	321
	9.4	Method of Good Lattice Points	325
	9.5	Estimating Quasi-Monte Carlo Errors	329
	9.6	Acceleration Methods and qMC Schemes	330
	9.7	Further Reading and Summary	330
		Exercises	331
ш	NUI	MERICAL METHODS FOR FUNCTIONAL PROBLEMS	333
10	Finit	e-Difference Methods	335
	10.1	Classification of Ordinary Differential Equations	335
	10.2	Solution of Linear Dynamic Systems	337
	10.3	Finite-Difference Methods for Initial Value Problems	340
	10.4	Economic Examples of IVPs	346
	10.5	Boundary Value Problems for ODEs: Shooting	350
	10 0		
	10.6	Finite-Horizon Optimal Control Problems	351
	10.6 $10.7$	Finite-Horizon Optimal Control Problems Infinite-Horizon Optimal Control and Reverse Shooting	351 355
	10.6 10.7 10.8	Finite-Horizon Optimal Control Problems Infinite-Horizon Optimal Control and Reverse Shooting Integral Equations	351 355 362
	10.6 10.7 10.8 10.9	Finite-Horizon Optimal Control Problems Infinite-Horizon Optimal Control and Reverse Shooting Integral Equations Further Reading and Summary	351 355 362 365
	10.6 10.7 10.8 10.9	Finite-Horizon Optimal Control Problems Infinite-Horizon Optimal Control and Reverse Shooting Integral Equations Further Reading and Summary Exercises	351 355 362 365 366
11	10.6 10.7 10.8 10.9 <b>Proj</b>	Finite-Horizon Optimal Control Problems Infinite-Horizon Optimal Control and Reverse Shooting Integral Equations Further Reading and Summary Exercises ection Methods for Functional Equations	351 355 362 365 366 369
11	10.6 10.7 10.8 10.9 <b>Proj</b> 11.1	Finite-Horizon Optimal Control Problems Infinite-Horizon Optimal Control and Reverse Shooting Integral Equations Further Reading and Summary Exercises ection Methods for Functional Equations An Ordinary Differential Equation Example	351 355 362 365 366 369 369
11	10.6 10.7 10.8 10.9 <b>Proj</b> 11.1 11.2	Finite-Horizon Optimal Control Problems Infinite-Horizon Optimal Control and Reverse Shooting Integral Equations Further Reading and Summary Exercises ection Methods for Functional Equations An Ordinary Differential Equation Example A Partial Differential Equation Example	351 355 362 365 366 369 369 375
11	10.6 10.7 10.8 10.9 <b>Proj</b> 11.1 11.2 11.3	Finite-Horizon Optimal Control Problems Infinite-Horizon Optimal Control and Reverse Shooting Integral Equations Further Reading and Summary Exercises ection Methods for Functional Equations An Ordinary Differential Equation Example A Partial Differential Equation Example General Projection Method	351 355 362 365 366 369 369 369 375 377

1.00			
-	Ĵ,	ntents	

	11.5	Continuous-Time Growth Model	392
	11.6	Computing Conditional Expectations	393
	11.7	Further Reading and Summary	395
		Exercises	396
12	Nume	rical Dynamic Programming	399
	12.1	Discrete-Time Dynamic Programming Problems	399
	12.2	Continuous-Time Dynamic Programming Problems	406
	12.3	Finite-State Methods	409
	12.4	Acceleration Methods for Infinite-Horizon Problems	415
	12.5	Discretization Methods for Continuous-State Problems	424
	12.6	Methods for Solving Linear-Quadratic Problems	431
	12.7	Continuous Methods for Continuous-State Problems	433
	12.8	Parametric Approximations and Simulation Methods	436
	12.9	Shape-Preserving Methods	437
	12.10	Continuous-Time Problems	440
	12.11	Further Reading and Summary	442
		Exercises	443
IV	PERT	URBATION METHODS	445
13	Regul	ar Perturbations of Simple Systems	447
	13.1	Mathematics of Regular Perturbation Methods	448
	13.1 13.2	Mathematics of Regular Perturbation Methods Comparative Statics	448 451
	13.1 13.2 13.3	Mathematics of Regular Perturbation Methods Comparative Statics Perturbing an IVP	448 451 453
	13.1 13.2 13.3 13.4	Mathematics of Regular Perturbation Methods Comparative Statics Perturbing an IVP Perturbing a BVP: Comparative Perfect Foresight Dynamics	448 451 453 456
	<ul> <li>13.1</li> <li>13.2</li> <li>13.3</li> <li>13.4</li> <li>13.5</li> </ul>	Mathematics of Regular Perturbation Methods Comparative Statics Perturbing an IVP Perturbing a BVP: Comparative Perfect Foresight Dynamics Continuous-Time Deterministic Control	448 451 453 456 462
	13.1 13.2 13.3 13.4 13.5 13.6	Mathematics of Regular Perturbation Methods Comparative Statics Perturbing an IVP Perturbing a BVP: Comparative Perfect Foresight Dynamics Continuous-Time Deterministic Control Stochastic Control	448 451 453 456 462 471
	13.1 13.2 13.3 13.4 13.5 13.6 13.7	Mathematics of Regular Perturbation Methods Comparative Statics Perturbing an IVP Perturbing a BVP: Comparative Perfect Foresight Dynamics Continuous-Time Deterministic Control Stochastic Control Perturbing Discrete-Time Systems	448 451 453 456 462 471 474
	13.1 13.2 13.3 13.4 13.5 13.6 13.7 13.8	Mathematics of Regular Perturbation Methods Comparative Statics Perturbing an IVP Perturbing a BVP: Comparative Perfect Foresight Dynamics Continuous-Time Deterministic Control Stochastic Control Perturbing Discrete-Time Systems Perturbing Jump Process Control Problems	448 451 453 456 462 471 474 480
	13.1 13.2 13.3 13.4 13.5 13.6 13.7 13.8 13.9	Mathematics of Regular Perturbation Methods Comparative Statics Perturbing an IVP Perturbing a BVP: Comparative Perfect Foresight Dynamics Continuous-Time Deterministic Control Stochastic Control Perturbing Discrete-Time Systems Perturbing Jump Process Control Problems Global Quality Test for Asymptotic Approximations	448 451 453 456 462 471 474 480 482
	<ul> <li>13.1</li> <li>13.2</li> <li>13.3</li> <li>13.4</li> <li>13.5</li> <li>13.6</li> <li>13.7</li> <li>13.8</li> <li>13.9</li> </ul>	Mathematics of Regular Perturbation Methods Comparative Statics Perturbing an IVP Perturbing a BVP: Comparative Perfect Foresight Dynamics Continuous-Time Deterministic Control Stochastic Control Perturbing Discrete-Time Systems Perturbing Jump Process Control Problems Global Quality Test for Asymptotic Approximations Exercises	448 451 453 456 462 471 474 480 482 484
14	13.1 13.2 13.3 13.4 13.5 13.6 13.7 13.8 13.9 <b>Regul</b>	Mathematics of Regular Perturbation Methods Comparative Statics Perturbing an IVP Perturbing a BVP: Comparative Perfect Foresight Dynamics Continuous-Time Deterministic Control Stochastic Control Perturbing Discrete-Time Systems Perturbing Jump Process Control Problems Global Quality Test for Asymptotic Approximations Exercises <b>ar Perturbations in Multidimensional Systems</b>	448 451 453 456 462 471 474 480 482 484 487
14	<ul> <li>13.1</li> <li>13.2</li> <li>13.3</li> <li>13.4</li> <li>13.5</li> <li>13.6</li> <li>13.7</li> <li>13.8</li> <li>13.9</li> </ul> Regular 14.1	Mathematics of Regular Perturbation Methods Comparative Statics Perturbing an IVP Perturbing a BVP: Comparative Perfect Foresight Dynamics Continuous-Time Deterministic Control Stochastic Control Perturbing Discrete-Time Systems Perturbing Jump Process Control Problems Global Quality Test for Asymptotic Approximations Exercises <b>ar Perturbations in Multidimensional Systems</b> Multidimensional Comparative Statics and Tensor Notation	448 451 453 456 462 471 474 480 482 484 487 487
14	13.1 13.2 13.3 13.4 13.5 13.6 13.7 13.8 13.9 <b>Regul</b> 14.1 14.2	Mathematics of Regular Perturbation Methods Comparative Statics Perturbing an IVP Perturbing a BVP: Comparative Perfect Foresight Dynamics Continuous-Time Deterministic Control Stochastic Control Perturbing Discrete-Time Systems Perturbing Jump Process Control Problems Global Quality Test for Asymptotic Approximations Exercises <b>ar Perturbations in Multidimensional Systems</b> Multidimensional Comparative Statics and Tensor Notation Linearization of Multidimensional Dynamic Systems	448 451 453 456 462 471 474 480 482 484 487 487 487 490
14	<ul> <li>13.1</li> <li>13.2</li> <li>13.3</li> <li>13.4</li> <li>13.5</li> <li>13.6</li> <li>13.7</li> <li>13.8</li> <li>13.9</li> </ul> Regulation 14.1 <ul> <li>14.2</li> <li>14.3</li> </ul>	Mathematics of Regular Perturbation Methods Comparative Statics Perturbing an IVP Perturbing a BVP: Comparative Perfect Foresight Dynamics Continuous-Time Deterministic Control Stochastic Control Perturbing Discrete-Time Systems Perturbing Jump Process Control Problems Global Quality Test for Asymptotic Approximations Exercises <b>ar Perturbations in Multidimensional Systems</b> Multidimensional Comparative Statics and Tensor Notation Linearization of Multidimensional Dynamic Systems Locally Asymptotically Stable Multidimensional Control	448 451 453 456 462 471 474 480 482 484 487 487 487 490 496

	14.4	Perturbations of Discrete-Time Problems	502
	14.5	Multisector Stochastic Growth	504
	14.6	Further Reading and Summary	509
		Exercises	509
15	Adva	nced Asymptotic Methods	511
	15.1	Bifurcation Methods	511
	15.2	Portfolio Choices for Small Risks	513
	15.3	Gauge Functions and Asymptotic Expansions	516
	15.4	Method of Undetermined Gauges	517
	15.5	Asymptotic Expansions of Integrals	522
	15.6	Hybrid Perturbation-Projection Methods	528
	15.7	Further Reading and Summary	532
		Exercises	533
V	APPI	LICATIONS TO DYNAMIC EQUILIBRIUM ANALYSIS	535
16	Soluti	ion Methods for Perfect Foresight Models	537
	16.1	A Simple Autonomous Overlapping Generations Model	538
	16.2	Equilibrium in OLG Models: Time Domain Methods	540
	16.3	Fair-Taylor Method	547
	16.4	Recursive Models and Dynamic Iteration Methods	549
	16.5	Recursive Models with Nonlinear Equation Methods	558
	16.6	Accuracy Measures	562
	16.7	Tax and Monetary Policy in Dynamic Economies	563
	16.8	Recursive Solution of an OLG Model	567
	16.9	"Consistent" Capital Income Taxation	568
	16.10	Further Reading and Summary	571
		Exercises	571
17	Solvir	ng Rational Expectations Models	573
	17.1	Lucas Asset-Pricing Model	574
	17.2	Monetary Equilibrium	577
	17.3	Information and Asset Markets	578
	17.4	Commodity Storage Models	581
	17.5	A Simple Stochastic Dynamic Growth Model	588
	17.6	Projection Methods with Newton Iteration	589

17.7	Fixed-Point Iteration	599
17.8	Time Iteration	601
17.9	Generalizations	602
17.10	Further Reading and Summary	605
	Exercises	606
Refere	ences	609
Index		623