
Contents

1. BASIC CONCEPTS

1.1	Introduction	1
1.2	Algebra of Events (Boolean Algebra)	3
1.3	Probability	10
1.4	Combinatorial Problems	15
1.5	Independence	25
1.6	Conditional Probability	33
1.7	Some Fallacies in Combinatorial Problems	39
1.8	Appendix: Stirling's Formula	43

2. RANDOM VARIABLES

2.1	Introduction	46
2.2	Definition of a Random Variable	48
2.3	Classification of Random Variables	51
2.4	Functions of a Random Variable	58
2.5	Properties of Distribution Functions	66
2.6	Joint Density Functions	70
2.7	Relationship Between Joint and Individual Densities; Independence of Random Variables	76
2.8	Functions of More Than One Random Variable	85
2.9	Some Discrete Examples	95

3. EXPECTATION

3.1	Introduction	100
3.2	Terminology and Examples	107
3.3	Properties of Expectation	114
3.4	Correlation	119
3.5	The Method of Indicators	122
3.6	Some Properties of the Normal Distribution	124
3.7	Chebyshev's Inequality and the Weak Law of Large Numbers	126

4. CONDITIONAL PROBABILITY AND EXPECTATION

4.1	Introduction	130
4.2	Examples	133
4.3	Conditional Density Functions	135
4.4	Conditional Expectation	140
4.5	Appendix: The General Concept of Conditional Expectation	152

5. CHARACTERISTIC FUNCTIONS

5.1	Introduction	154
5.2	Examples	158
5.3	Properties of Characteristic Functions	166
5.4	The Central Limit Theorem	169

6. INFINITE SEQUENCES OF RANDOM VARIABLES

6.1	Introduction	178
6.2	The Gambler's Ruin Problem	182
6.3	Combinatorial Approach to the Random Walk; the Reflection Principle	186
6.4	Generating Functions	191
6.5	The Poisson Random Process	196
6.6	The Strong Law of Large Numbers	203

7. MARKOV CHAINS

7.1	Introduction	211
7.2	Stopping Times and the Strong Markov Property	217
7.3	Classification of States	220
7.4	Limiting Probabilities	230
7.5	Stationary and Steady-State Distributions	236

8. INTRODUCTION TO STATISTICS

8.1	Statistical Decisions	241
8.2	Hypothesis Testing	243
8.3	Estimation	258
8.4	Sufficient Statistics	264
8.5	Unbiased Estimates Based on a Complete Sufficient Statistic	268
8.6	Sampling from a Normal Population	274
8.7	The Multidimensional Gaussian Distribution	279

<i>Tables</i>	286
<i>A Brief Bibliography</i>	289
<i>Solutions to Problems</i>	290
<i>Index</i>	333